Problem A

Phone List

Given a list of phone numbers, determine if it is consistent in the sense that no number is the prefix of another. Let's say the phone catalogue listed these numbers:

- Emergency 911
- Alice 97 625 999
- Bob 91 12 54 26

In this case, it's not possible to call Bob, because the central would direct your call to the emergency line as soon as you had dialled the first three digits of Bob's phone number. So this list would not be consistent.

Input specifications

The first line of input gives a single integer, $1 \leq t \leq 40$, the number of test cases. Each test case starts with n, the number of phone numbers, on a separate line, $1 \leq n \leq 10000$. Then follows n lines with one unique phone number on each line. A phone number is a sequence of at most ten digits.

Output specifications

For each test case, output "YES" if the list is consistent, or "NO" otherwise.

Sample input

```
2
3
911
97625999
91125426
5
113
12340
123440
12345
98346
```

Output for sample input

```
NO
YES
```

Problem B

Cuckoo Hashing

One of the most fundamental data structure problems is the dictionary problem: given a set D of words you want to be able to quickly determine if any given query string q is present in the dictionary D or not. Hashing is a well-known solution for the problem. The idea is to create a function $h : \Sigma^* \rightarrow [0..n-1]$ from all strings to the integer range $0, 1, .., n-1$, i.e. you describe a fast deterministic program which takes a string as input and outputs an integer between 0 and $n-1$. Next you allocate an empty hash table T of size n and for each word w in D, you set $T[h(w)] = w$. Thus, given a query string q, you only need to calculate $h(q)$ and see if $T[h(q)]$ equals q, to determine if q is in the dictionary. Seems simple enough, but aren't we forgetting something? Of course, what if two words in D map to the same location in the table? This phenomenon, called collision, happens fairly often (remember the Birthday paradox: in a class of 24 pupils there is more than 50% chance that two of them share birthday). On average you will only be able to put roughly $\sqrt{n}$-sized dictionaries into the table without getting collisions, quite poor space usage!

A stronger variant is Cuckoo Hashing[1]. The idea is to use two hash functions h_1 and h_2. Thus each string maps to two positions in the table. A query string q is now handled as follows: you compute both $h_1(q)$ and $h_2(q)$, and if $T[h_1(q)] = q$, *or* $T[h_2(q)] = q$, you conclude that q is in D. The name "Cuckoo Hashing" stems from the process of creating the table. Initially you have an empty table. You iterate over the words d in D, and insert them one by one. If $T[h_1(d)]$ is free, you set $T[h_1(d)] = d$. Otherwise if $T[h_2(d)]$ is free, you set $T[h_2(d)] = d$. If both are occupied however, just like the cuckoo with other birds' eggs, you evict the word r in $T[h_1(d)]$ and set $T[h_1(d)] = d$. Next you put r back into the table in its alternative place (and if that entry was already occupied you evict that word and move it to its alternative place, and so on). Of course, we may end up in an infinite loop here, in which case we need to rebuild the table with other choices of hash functions. The good news is that this will not happen with great probability even if D contains up to $n/2$ words!

Input specifications

On the first line of input is a single positive integer $1 \leq t \leq 50$ specifying the number of test cases to follow. Each test case begins with two positive integers $1 \leq m \leq n \leq 10000$ on a line of itself, m telling the number of words in the dictionary and n the size of the hash table in the test case. Next follow m lines of which the i:th describes the i:th word d_i in the dictionary D by two non-negative integers $h_1(d_i)$ and $h_2(d_i)$ less than n giving

[1]Cuckoo Hashing was suggested by the danes R. Pagh and F. F. Rödler in 2001

the two hash function values of the word d_i. The two values may be identical.

Output specifications

For each test case there should be exactly one line of output either containing the string "`successful hashing`" if it is possible to insert all words in the given order into the table, or the string "`rehash necessary`" if it is impossible.

Sample input

```
2
3 3
0 1
1 2
2 0
5 6
2 3
3 1
1 2
5 1
2 5
```

Output for sample input

```
successful hashing
rehash necessary
```

Problem C

Optimal Parking

When shopping on Long Street, Michael usually parks his car at some random location, and then walks to the stores he needs. Can you help Michael choose a place to park which minimises the distance he needs to walk on his shopping round?

Long Street is a straight line, where all positions are integer. You pay for parking in a specific slot, which is an integer position on Long Street. Michael does not want to pay for more than one parking though. He is very strong, and does not mind carrying all the bags around.

Input specifications

The first line of input gives the number of test cases, $1 \leq t \leq 100$. There are two lines for each test case. The first gives the number of stores Michael wants to visit, $1 \leq n \leq 20$, and the second gives their n integer positions on Long Street, $0 \leq x_i \leq 99$.

Output specifications

Output for each test case a line with the minimal distance Michael must walk given optimal parking.

Sample input

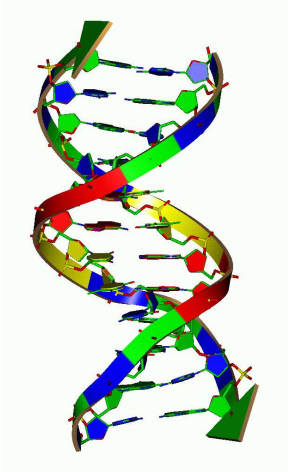
```
2
4
24 13 89 37
6
7 30 41 14 39 42
```

Output for sample input

```
152
70
```

Problem D

Copying DNA



Evolution is a seemingly random process which works in a way which resembles certain approaches we use to get approximate solutions to hard combinatorial problems. You are now to do something completely different.

Given a DNA string S from the alphabet {`A`,`C`,`G`,`T`}, find the minimal number of copy operations needed to create another string T. You may reverse the strings you copy, and copy both from S and the pieces of your partial T. You may put these pieces together at any time. You may only copy contiguous parts of your partial T, and all copied strings must be used in your final T. Example: From S = "`ACTG`" create T = "`GTACTATTATA`"

1. Get `GT.........` by copying and reversing "`TG`" from S.
2. Get `GTAC.......` by copying "`AC`" from S.
3. Get `GTAC...TA..` by copying "`TA`" from the partial T.
4. Get `GTAC...TAAT` by copying and reversing "`TA`" from the partial T.
5. Get `GTACAATTAAT` by copying "`AAT`" from the partial T.

Input specifications

The first line of input gives a single integer, $1 \leq t \leq 100$, the number of test cases. Then follow, for each test case, a line with the string S of length $1 \leq m \leq 18$, and a line with the string T of length $1 \leq n \leq 18$.

Output specifications

Output for each test case the number of copy operations needed to create T from S, or "`impossible`" if it cannot be done.

Sample input

```
5
ACGT
GTAC
A
C
ACGT
TGCA
ACGT
TCGATCGA
A
AAAAAAAAAAAAAAAAA
```

Output for sample input

```
2
impossible
1
4
6
```

Problem E

Circle of Debt

The three friends Alice, Bob, and Cynthia always seem to get in situations where there are debts to be cleared among themselves. Of course, this is the "price" of hanging out a lot: it only takes a few resturant visits, movies, and drink rounds to get an unsettled balance. So when they meet as usual every Friday afternoon they begin their evening by clearing last week's debts. To satisfy their mathematically inclined minds they prefer clearing their debts using as little money transaction as possible, i.e. by exchanging as few bank notes and coins as necessary. To their surprise, this can sometimes by harder than it sounds. Suppose that Alice owes Bob 10 crowns and this is the three friends' only uncleared debt, and Alice has a 50 crown note but nothing smaller, Bob has three 10 crown coins and ten 1 crown coins, and Cynthia has three 20 crown notes. The best way to clear the debt is for Alice to give her 50 crown note to Cynthia, Cynthia to give two 20 crown notes to Alice and one to Bob, and Bob to give one 10 crown coin to Cynthia, involving a total of only five notes/coins changing owners. Compare this to the straightforward solution of Alice giving her 50 crown note to Bob and getting Bob's three 10 crown notes and all his 1 crown coins for a total of fourteen notes/coins being exchanged!

Input specifications

On the first line of input is a single positive integer, $1 \leq t \leq 50$, specifying the number of test cases to follow. Each test case begins with three integers $ab, bc, ca \leq 1000$ on a line of itself. ab is the amount Alice owes Bob (negative if it is Bob who owes Alice money), bc the amount Bob owes Cynthia (negative if it is Cynthia who is in debt to Bob), and ca the amount Cynthia owes Alice (negative if it is Alice who owes Cynthia).

Next follow three lines each with six non-negative integers $a_{100}, a_{50}, a_{20}, a_{10}, a_5, a_1$, $b_{100}, \ldots, b_1$, and $c_{100}, \ldots c_1$, respectively, where a_{100} is the number of 100 crown notes Alice got, a_{50} is the number of her 50 crown notes, and so on. Likewise, $b_{100}, \ldots, b_1$ is the amount of notes/coins of different value Bob got, and $c_{100}, \ldots, c_1$ describes Cynthia's money. Each of them has at most 30 coins (i.e. $a_{10}+a_5+a_1, b_{10}+b_5+b_1$, and $c_{10}+c_5+c_1$ are all less than or equal to 30) and the total amount of all their money together (Alice's plus Bob's plus Cynthia's) is always less than 1000 crowns.

Output specifications

For each test case there should be one line of output containing the minimum number of bank notes and coins needed to settle the balance. If it is not possible at all, output the string "`impossible`".

Sample input

```
3
10 0 0
0 1 0 0 0 0
0 0 0 3 0 10
0 0 3 0 0 0
-10 -10 -10
0 0 0 0 0 0
0 0 0 0 0 0
0 0 0 0 0 0
-10 10 10
3 0 0 0 2 0
0 2 0 0 0 1
0 0 1 1 0 3
```

Output for sample input

```
5
0
impossible
```

Problem F

Full Tank?

After going through the receipts from your car trip through Europe this summer, you realised that the gas prices varied between the cities you visited. Maybe you could have saved some money if you were a bit more clever about where you filled your fuel?

To help other tourists (and save money yourself next time), you want to write a program for finding the cheapest way to travel between cities, filling your tank on the way. We assume that all cars use one unit of fuel per unit of distance, and start with an empty gas tank.

Input specifications

The first line of input gives $1 \leq n \leq 1000$ and $0 \leq m \leq 10000$, the number of cities and roads. Then follows a line with n integers $1 \leq p_i \leq 100$, where p_i is the fuel price in the ith city. Then follow m lines with three integers $0 \leq u, v < n$ and $1 \leq d \leq 100$, telling that there is a road between u and v with length d. Then comes a line with the number $1 \leq q \leq 100$, giving the number of queries, and q lines with three integers $1 \leq c \leq 100$, s and e, where c is the fuel capacity of the vehicle, s is the starting city, and e is the goal.

Output specifications

For each query, output the price of the cheapest trip from s to e using a car with the given capacity, or "`impossible`" if there is no way of getting from s to e with the given car.

Sample input

```
5 5
10 10 20 12 13
0 1 9
0 2 8
1 2 1
1 3 11
2 3 7
2
10 0 3
20 1 4
```

Output for sample input

```
170
impossible
```

Problem G

Nested Dolls

Dilworth is the world's most prominent collector of Russian nested dolls: he literally has thousands of them! You know, the wooden hollow dolls of different sizes of which the smallest doll is contained in the second smallest, and this doll is in turn contained in the next one and so forth. One day he wonders if there is another way of nesting them so he will end up with fewer nested dolls? After all, that would make his collection even more magnificent! He unpacks each nested doll and measures the width and height of each contained doll. A doll with width w_1 and height h_1 will fit in another doll of width w_2 and height h_2 if and only if $w_1 < w_2$ and $h_1 < h_2$. Can you help him calculate the smallest number of nested dolls possible to assemble from his massive list of measurements?

Input specifications

On the first line of input is a single positive integer $1 \leq t \leq 20$ specifying the number of test cases to follow. Each test case begins with a positive integer $1 \leq m \leq 20000$ on a line of itself telling the number of dolls in the test case. Next follow $2m$ positive integers $w_1, h_1, w_2, h_2, \ldots, w_m, h_m$, where w_i is the width and h_i is the height of doll number i. $1 \leq w_i, h_i \leq 10000$ for all i.

Output specifications

For each test case there should be one line of output containing the minimum number of nested dolls possible.

Sample input

```
4
3
20 30 40 50 30 40
4
20 30 10 10 30 20 40 50
3
10 30 20 20 30 10
4
10 10 20 30 40 50 39 51
```

Output for sample input

```
1
2
3
2
```

Problem H

Shopaholic

Lindsay is a shopaholic. Whenever there is a discount of the kind where you can buy three items and only pay for two, she goes completely mad and feels a need to buy all items in the store. You have given up on curing her for this disease, but try to limit its effect on her wallet.

You have realized that the stores coming with these offers are quite selective when it comes to which items you get for free; it is always the cheapest ones. As an example, when your friend comes to the counter with seven items, costing 400, 350, 300, 250, 200, 150, and 100 dollars, she will have to pay 1500 dollars. In this case she got a discount of 250 dollars. You realize that if she goes to the counter three times, she might get a bigger discount. E.g. if she goes with the items that costs 400, 300 and 250, she will get a discount of 250 the first round. The next round she brings the item that costs 150 giving no extra discount, but the third round she takes the last items that costs 350, 200 and 100 giving a discount of an additional 100 dollars, adding up to a total discount of 350.

Your job is to find the maximum discount Lindsay can get.

Input specifications

The first line of input gives the number of test scenarios, $1 \leq t \leq 20$. Each scenario consists of two lines of input. The first gives the number of items Lindsay is buying, $1 \leq n \leq 20000$. The next line gives the prices of these items, $1 \leq p_i \leq 20000$.

Output specifications

For each scenario, output one line giving the maximum discount Lindsay can get by selectively choosing which items she brings to the counter at the same time.

Sample input

```
1
6
400 100 200 350 300 250
```

Output for sample input

```
400
```

Problem I

Moogle

You got the original idea of making map software, called Moogle Maps, for the new cool Maple mPhone. It will even be capable of indicating the location of a house address like "Main Street 13". However, since the mPhone has limited storage capacity, you need to reduce the data amount. You don't want to store the exact location of every single house number. Instead only a subset of the house numbers will be stored exactly, and the others will be linearly interpolated. So you want to select house numbers that will minimise the average interpolation error, given how many house locations you have capacity to store. We view the street as a straight line, and you will always store the first and the last house location.

Given that you've stored the locations x_i and x_j for the houses with numbers i and j respectively, but no other house in between, the interpolated value for a house with number k with $i < k < j$ is $x_i + (x_j - x_i) \cdot \frac{k-i}{j-i}$.

Input specifications

The first line of input gives a single integer, $1 \leq t \leq 50$, the number of test cases.

For each test case, there are two lines. The first contains $2 \leq h \leq 200$ and $2 \leq c \leq h$, where h is the number of houses in the street and c is the number of house locations that can be stored. The second contains h integers in increasing order giving the location of the h houses. Each location is in the interval $[0, 1000000]$.

Output specifications

For each test case, output the average interpolation error over all the h houses for the optimal selection of c house locations to store. The output should be given with four decimal places, but we will accept inaccuracies of up to ± 0.001.

Sample input

```
2
4 3
0 9 20 40
10 4
0 10 19 30 40 90 140 190 202 210
```

Output for sample input

```
0.2500
0.3000
```

Problem J: GPA

Source: `gpa.{c,cpp,java}`
Input: `gpa.in`
Output: `gpa.out`

Each course grade is one of the following five letters: A, B, C, D, and F. (Note that there is no grade E.) The grade A indicates superior achievement, whereas F stands for failure. In order to calculate the GPA, the letter grades A, B, C, D, and F are assigned the following grade points, respectively: 4, 3, 2, 1, and 0.

Input

The input file will contain data for one or more test cases, one test case per line. On each line there will be one or more upper case letters, separated by blank spaces.

Output

Each line of input will result in exactly one line of output. If all upper case letters on a particular line of input came from the set {A, B, C, D, F} then the output will consist of the GPA, displayed with a precision of two decimal places. Otherwise, the message "Unknown letter grade in input" will be printed.

Sample input

```
A B C D F
B F F C C A
D C E F
```

Output for sample input

```
2.00
1.83
Unknown letter grade in input
```

Problem K: Lawrence of Arabia

Source: `lawrence.{c,cpp,java}`
Input: `lawrence.in`
Output: `lawrence.out`

T. E. Lawrence was a controversial figure during World War I. He was a British officer who served in the Arabian theater and led a group of Arab nationals in guerilla strikes against the Ottoman Empire. His primary targets were the railroads. A highly fictionalized version of his exploits was presented in the blockbuster movie, "Lawrence of Arabia".

You are to write a program to help Lawrence figure out how to best use his limited resources. You have some information from British Intelligence. First, the rail line is completely linear---there are no branches, no spurs. Next, British Intelligence has assigned a Strategic Importance to each depot---an integer from 1 to 5. A depot is of no use on its own, it only has value if it is connected to other depots. The Strategic Value of the entire railroad is calculated by adding up the products of the Strategic Values for every pair of depots that are connected, directly or indirectly, by the rail line. Consider this railroad:

Its Strategic Value is 4*5 + 4*1 + 4*2 + 5*1 + 5*2 + 1*2 = 49.

Now, suppose that Lawrence only has enough resources for one attack. He cannot attack the depots themselves---they are too well defended. He must attack the rail line between depots, in the middle of the desert. Consider what would happen if Lawrence attacked this rail line right in the middle:

The Strategic Value of the remaining railroad is 4*5 + 1*2 = 22. But, suppose Lawrence attacks between the 4 and 5 depots:

The Strategic Value of the remaining railroad is 5*1 + 5*2 + 1*2 = 17. This is Lawrence's best option.

Given a description of a railroad and the number of attacks that Lawrence can perform, figure out the smallest Strategic Value that he can achieve for that railroad.

Input

There will be several data sets. Each data set will begin with a line with two integers, n and m. n is the number of depots on the railroad ($1 \leq n \leq 1000$), and m is the number of attacks Lawrence has resources for ($0 \leq m < n$). On the next line will be n integers, each from 1 to 5, indicating the Strategic Value of each depot in order. End of input will be marked by a line with $n=0$ and $m=0$, which should not be processed.

Output

For each data set, output a single integer, indicating the smallest Strategic Value for the railroad that Lawrence can achieve with his attacks. Output each integer in its own line.

Sample Input

```
4 1
4 5 1 2
4 2
4 5 1 2
0 0
```

Sample Output

```
17
2
```

Problem L: Walk in the Park

Source: `park.{c,cpp,java}`
Input: `park.in`
Output: `park.out`

You are responsible for inspecting trees located in a park, to make sure they remain healthy. The location of each tree is given to you as a point in the two-dimensional plane. Due to recently-replanted grass, you are only allowed to walk through the park along a collection of paths. Each path is described by an infinite-length horizontal or vertical line in the two-dimensional plane.

You are concerned that it may not be possible to view all the trees in the park from some vantage point on a path. In particular, a tree is visible only if you can view it by standing on some path while facing perpendicular to the path. There must be no intervening tree that obstructs your view.

Input

The input file contains a single park configuration in the form:

```
NTREES NPATHS
X(1) Y(1)
.
.
.
X(NTREES) Y(NTREES)
PATH(1)
.
.
.
PATH(NPATHS)
```

`NTREES` and **`NPATHS`** are integers in the range [1,100000]. The integer coordinates of the trees are given on the next **`NTREES`** lines. This is followed by **`NPATHS`** lines, each of the form **`x=C`** or **`y=C`** defining a vertical or horizontal path, where C is an integer in the range [-1000000,1000000].

All coordinates are in the range $-1000000 \leq x,y \leq 1000000$. All trees are distinct and do not lie on any path, and all paths are distinct.

Output

The number of trees visible from some path.

Sample Input

```
6 3
-1 3
4 2
6 2
6 3
6 4
4 3
x=0
y=-1
y=5
```

Sample Output

```
5
```